ROC Curves for Methods of Evaluating Evidence:

Common Performance Measures Based on Similarity Scores

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Outline

- Introduction
- Background
- Analysis
- Results
- Conclusion

INTRODUCTION

Context

- Two samples
  - Control
  - Recovered
- Two hypotheses
  - Same source
  - Different sources

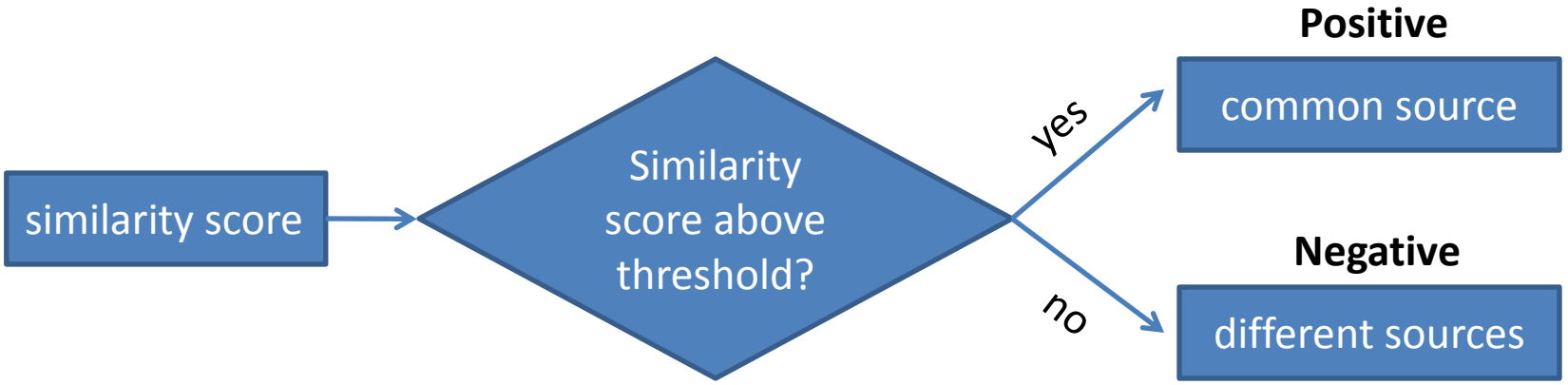
Evidence Evaluation

Similarity Score

- Numerical
- Indicative of association
- Higher values more suggestive of common source

Thresholds and Errors

- Threshold: fixed cutoff on similarity scores



- Method evaluation data: known sources
- Error types:
  - False positive
  - False negative

Outcomes for Fixed Threshold

		Truth	
		positive: pairs from same source	negative: pairs from different source
Evidence evaluation method's indication	positive	true positives	false positives
	negative	false negatives	true negatives

Error Rates

1. False positive rate = $$\frac{\text{number of false positives}}{\text{number of true negatives} + \text{number of false positives}}$$

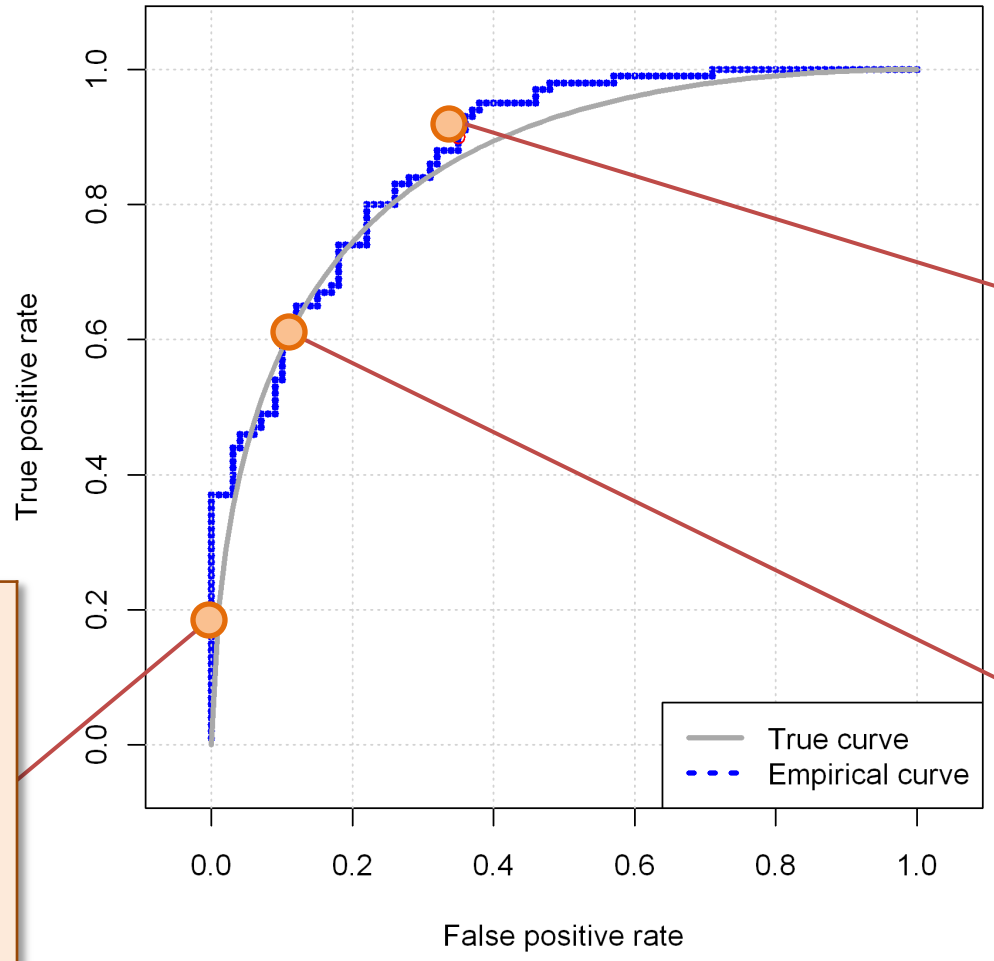
2. False negative rate = $$\frac{\text{number of false negatives}}{\text{number of true positives} + \text{number of false negatives}}$$

Or,

True positive rate = $$\frac{\text{number of true positives}}{\text{number of true positives} + \text{number of false negatives}}$$

Outcomes for Varied Threshold

Receiver operating characteristic (ROC) plot



Threshold = 0.37

Indication		Truth p	Truth n
	p	90	35
	n	10	65

Threshold = 2.37

Indication		Truth p	Truth n
	p	20	0
	n	80	100

Threshold = 1.16

Indication		Truth p	Truth n
	p	60	10
	n	40	90

Application to Forensics

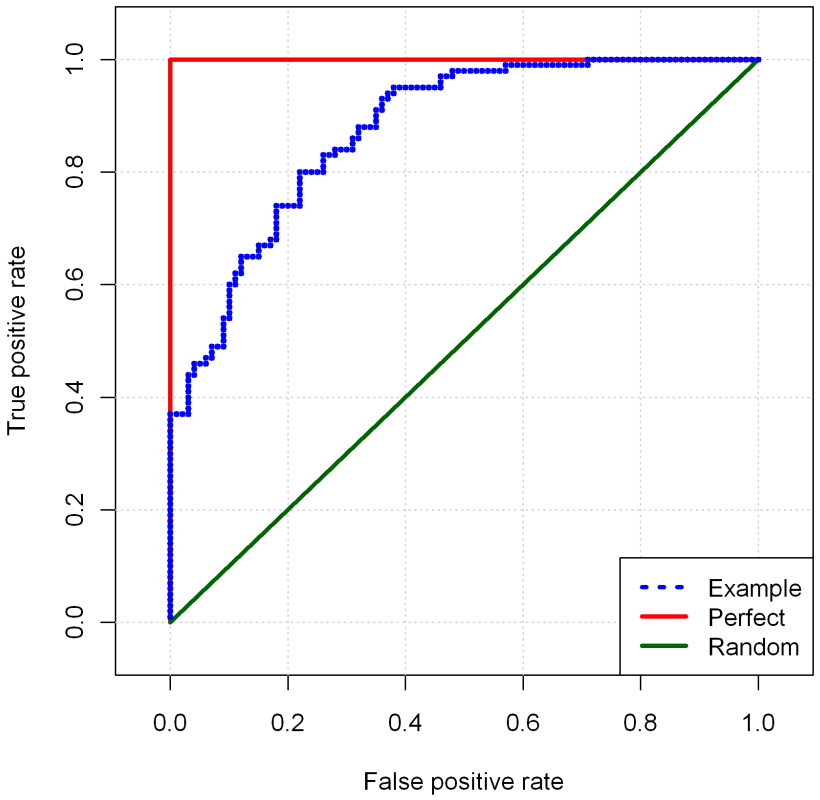
- Glass fragments
- Statistical methods of evaluating evidence

BACKGROUND

History and Uses of ROC Curves

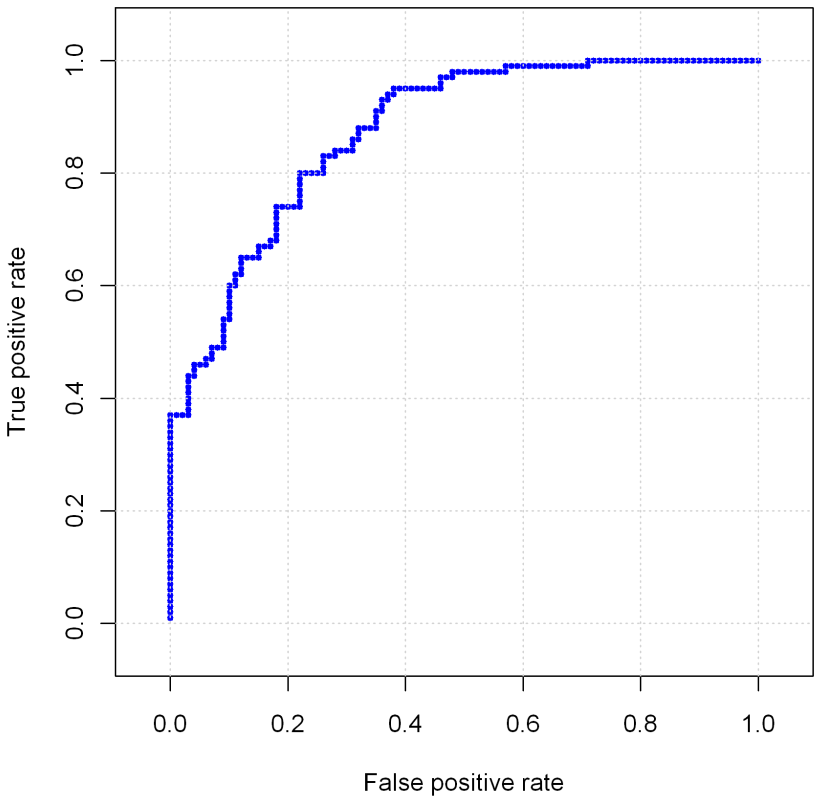
- 1940s: Radar hits and misses
- 1950s and 1960s: Signal detection theory (Green and Swets, 1966)
- 1980s: Diagnostic systems (Swets and Pickett, 1982)
- Today: Medicine, machine learning, astronomy and more

Overview of ROC Curves



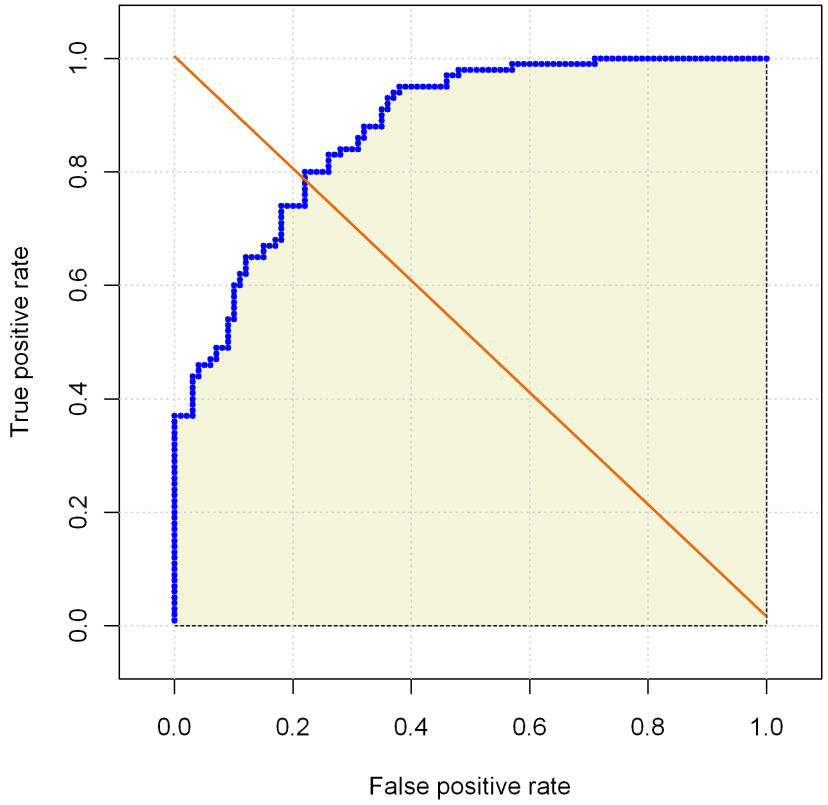
- Axes
  - True positive rate vs false positive rate
  - Range from 0 to 1
- Possible ROC curves
  - Perfect from (0,0) to (0,1) to (1,1).
  - Random along 45 degree line
  - In practice, usually somewhere between

Properties of ROC Curves



- Complete range of error rates
- Independent of scale of similarity scores
  - Order of similarity scores determines curve
  - Invariant under non-decreasing monotone transformation

ROC Performance Measures



- Error rates (upper left region)
- Equal error rate
- Equal likelihood
- Area under the curve (AUC)

$$= \Pr(\text{score}_{\text{same}} > \text{score}_{\text{diff}})$$

ANALYSIS

Glass data

- 62 windows
  - Three types
- Five fragments from each window
- Measurements of Si, K, Ca, and Fe
- Variables: log(Ca/K), log(Ca/Si), log(Ca/Fe)

Aitken, C. G. G., and Lucy, D. (January 2004). Evaluation of trace evidence in the form of multivariate data. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 53 (1), 109-122.

Data publicly available at
http://www.blackwellpublishing.com/rss/

Statistical Methods of Evaluating Evidence

- Methods (Aitken and Lucy, 2004):
  - Multiple *t*-statistics
  - Hotelling's T^2-statistic
  - Normal-based likelihood ratio
  - Density-based likelihood ratio
- Similarity scores:

  We can treat all methods as mappings from two samples to a similarity score.

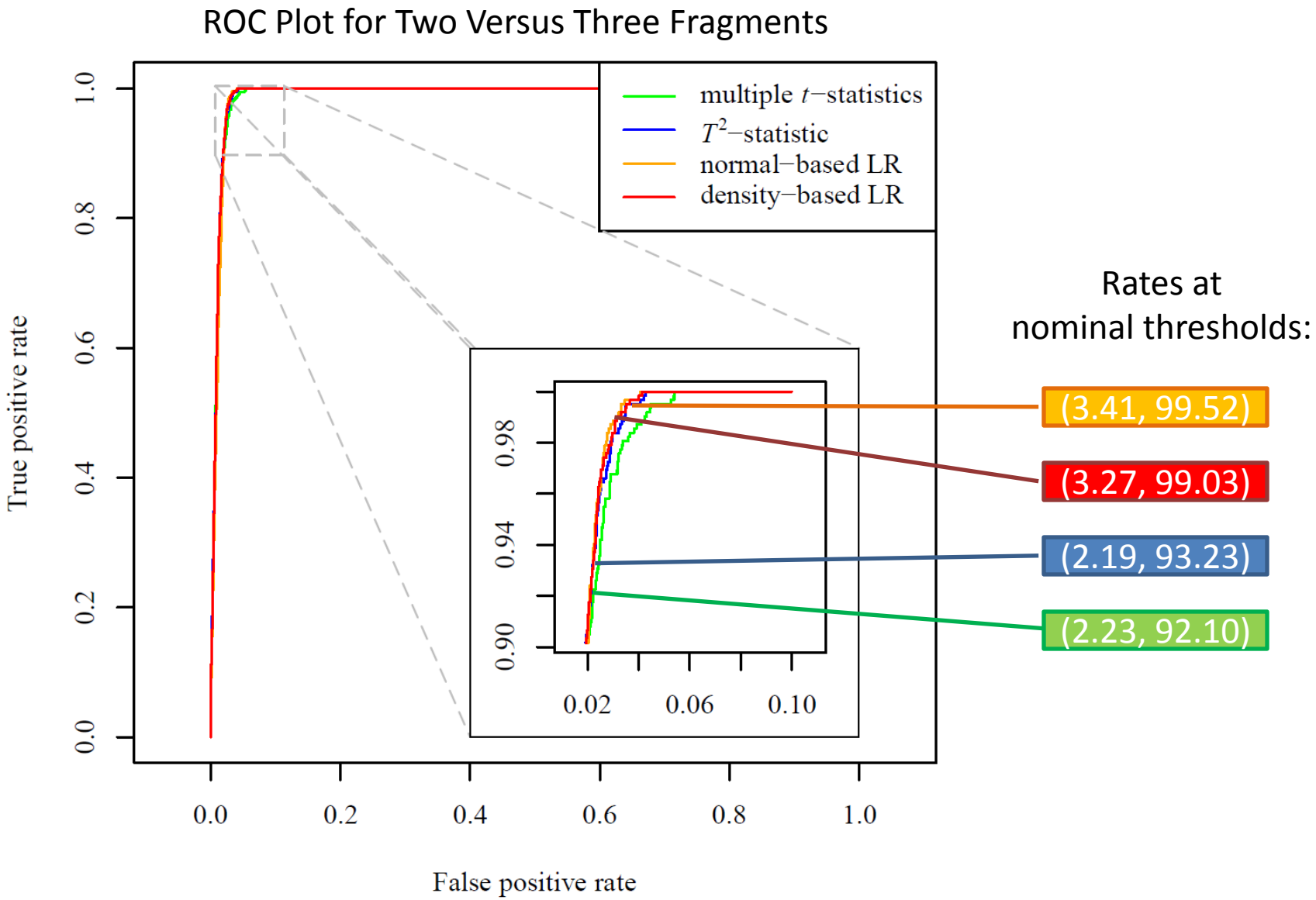
RESULTS

Nominal Error Rates

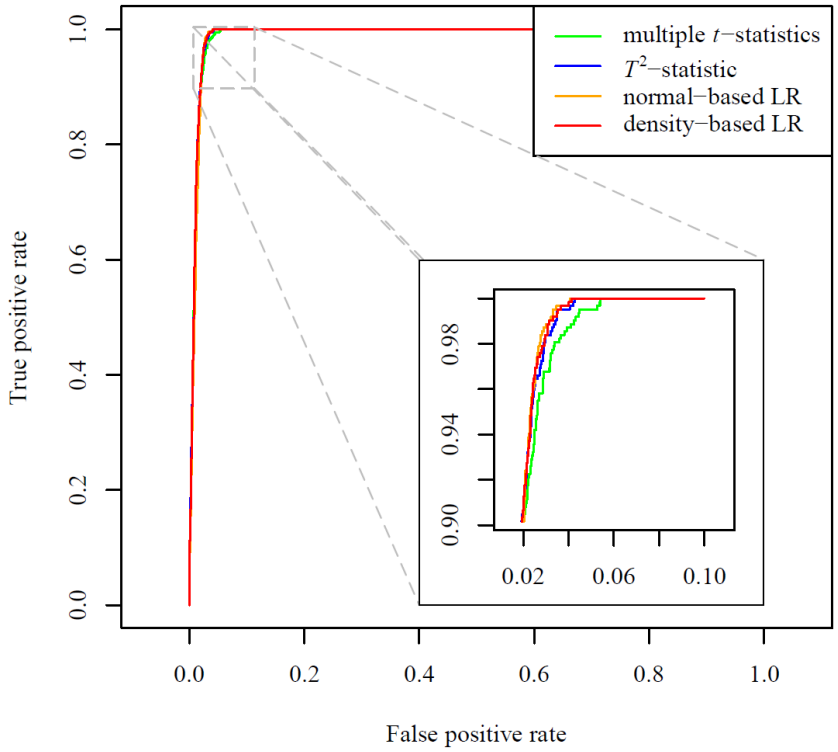
Error rates at nominal thresholds for two versus three fragments:

Error type	Multiple *t*-statistics	T^2-statistic	Normal-based LR	Density-based LR
false negative	7.90	6.77	0.48	0.97
false positive	2.23	2.19	3.41	3.27

ROC Plots


ROC Plot for Two Versus Three Fragments
multiple t−statistics
T^2−statistic
normal−based LR
density−based LR
True positive rate
False positive rate
0.0
0.2
0.4
0.6
0.8
1.0
0.90
0.94
0.98
0.02
0.06
0.10
Rates at nominal thresholds:
(3.41, 99.52)
(3.27, 99.03)
(2.19, 93.23)
(2.23, 92.10)

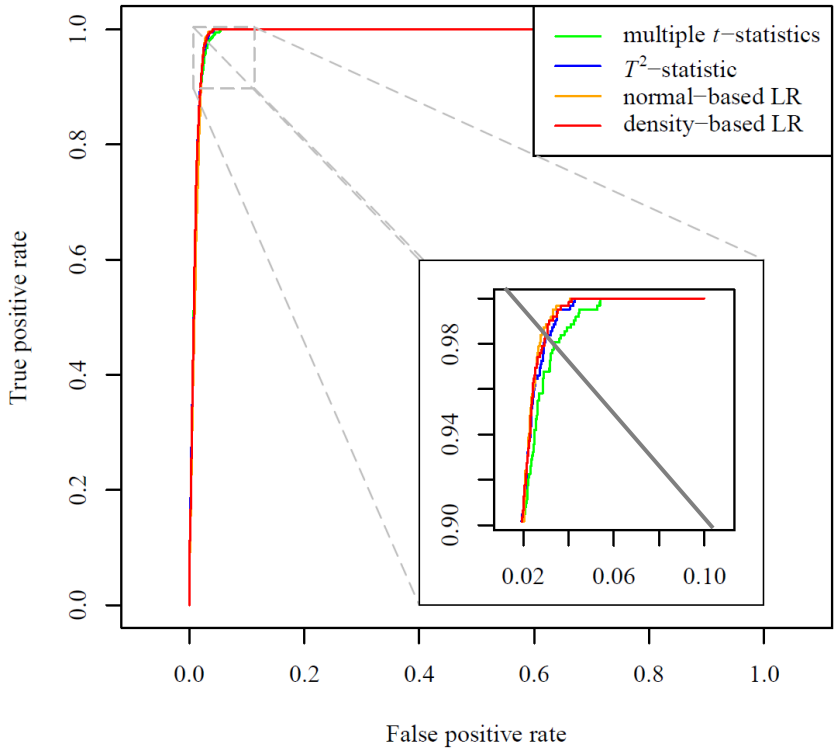
AUC Values



AUC values

Method	AUC
Multiple *t*-statistics	99.03
T^2-statistic	99.08
Normal-based LR	98.98
Density-based LR	99.09

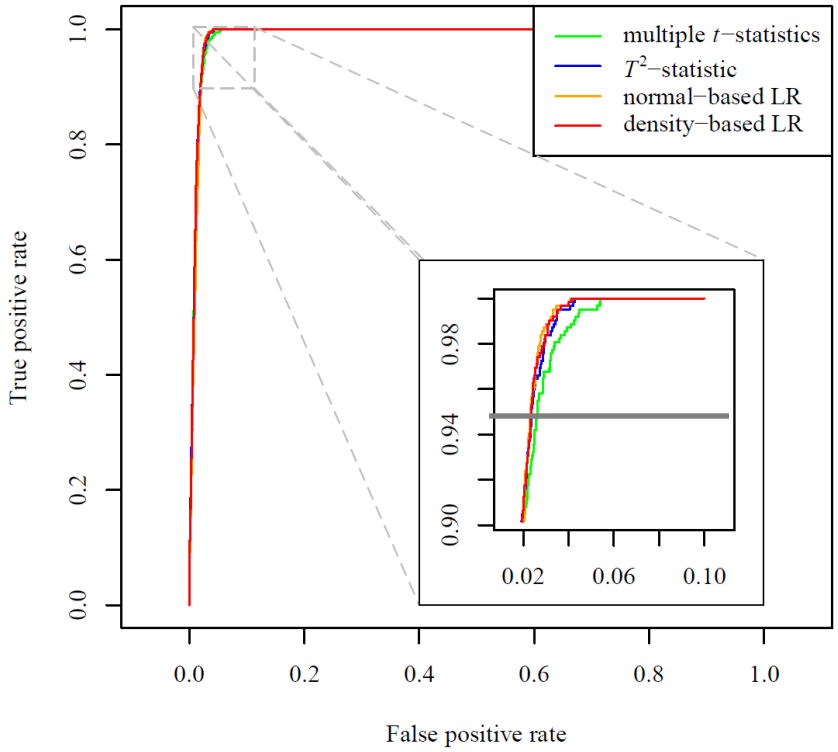
EER Values



EER values

Method	EER	Threshold
Multiple *t*-statistics	3.23	3.29
T^2-statistic	2.81	12.44
Normal-based LR	2.61	38.32
Density-based LR	2.62	17.98

5% False Negative Rate



False Positive Rates

Method	False Positive Rate	Threshold
Multiple *t*-statistics	2.61	2.72
T^2-statistic	2.36	8.96
Normal-based LR	2.30	117.33
Density-based LR	2.35	44.71

CONCLUSION

Conclusions

- ROC curves from similarity scores are
  - Comprehensive
    - Full range of error rates
  - Comparable
    - Independent of scale of scores
    - Objective performance measures

- Application to trace evidence and statistical methods (glass data) showed high performance from all methods.

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