ROC Curves for Methods of Evaluating Evidence:

Common Performance Measures Based on Similarity Scores

R. Bradley Patterson, Department of Statistics, George Mason University
John Miller, Department of Statistics, George Mason University
Chris Saunders, Department of Applied IT, George Mason University

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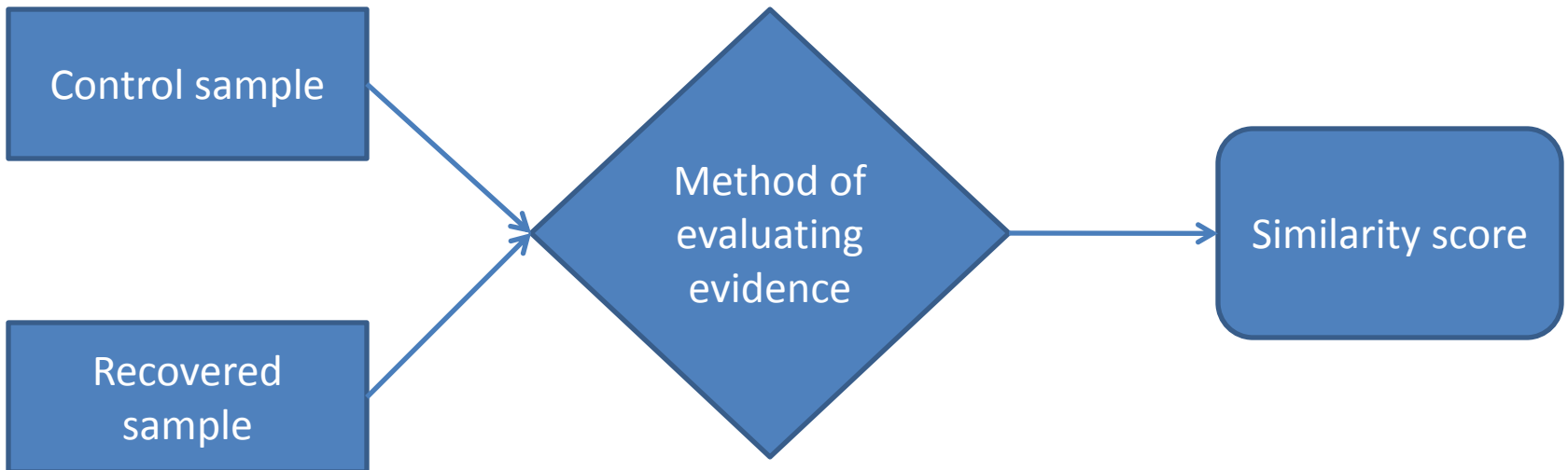# Acknowledgement and Disclaimer

Outline

- Introduction

- Background

- Analysis

- Results

- Conclusion

INTRODUCTION

Context

- Two samples
  - Control
  - Recovered
- Two hypotheses
  - Same source
  - Different sources
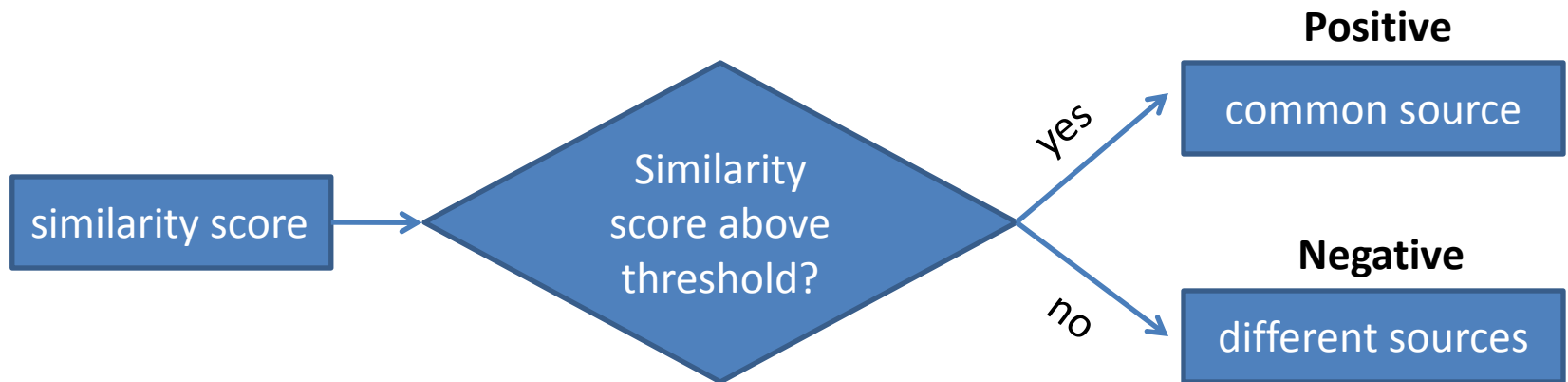
Evidence Evaluation

Similarity Score

- Numerical
- Indicative of association
- Higher values more suggestive of common source

Thresholds and Errors

- Threshold: fixed cutoff on similarity scores

- Method evaluation data: known sources
- Error types:
  - False positive
  - False negative

Outcomes for Fixed Threshold

Truth

		positive: pairs from same source	negative: pairs from different source
Evidence evaluation method's indication	positive	true positives	false positives
	negative	false negatives	true negatives

Error Rates

1. False positive rate = $\dfrac{\text{number of false positives}}{\text{number of true negatives} + \text{number of false positives}}$
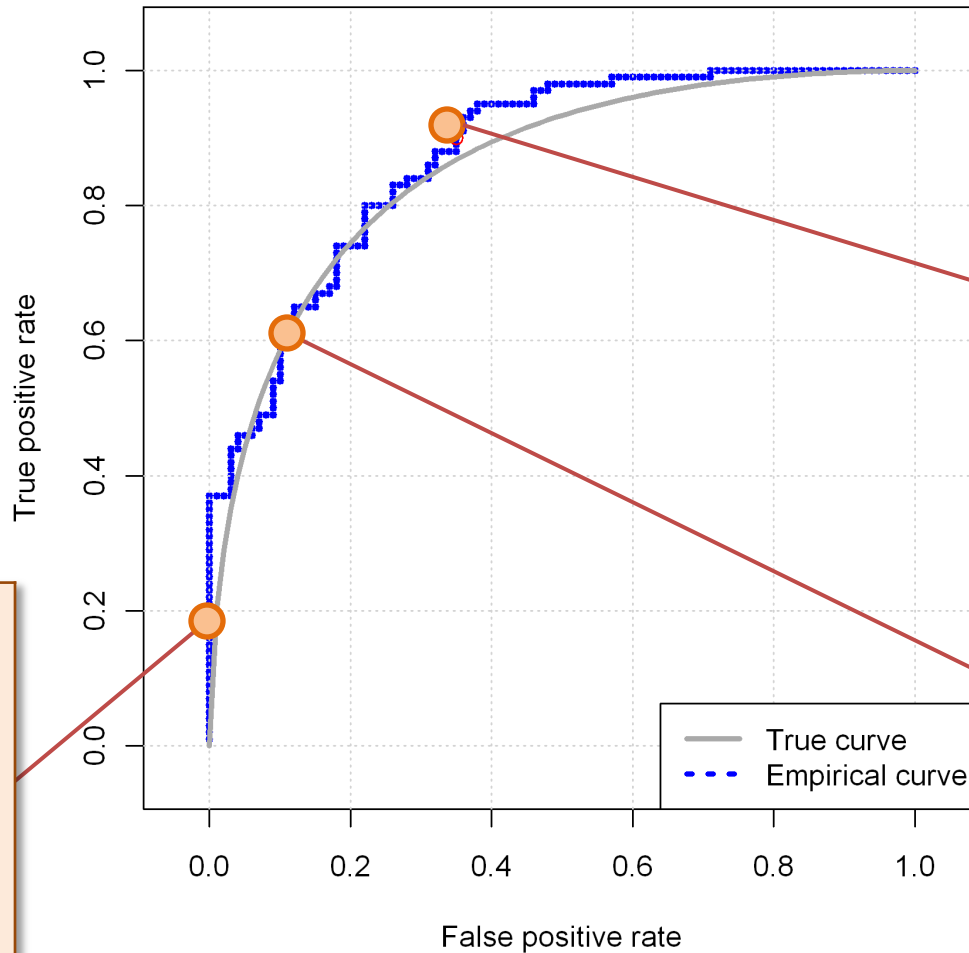
2. False negative rate = $\dfrac{\text{number of false negatives}}{\text{number of true positives} + \text{number of false negatives}}$

Or,

True positive rate = $\dfrac{\text{number of true positives}}{\text{number of true positives} + \text{number of false negatives}}$

Outcomes for Varied Threshold

Receiver operating characteristic (ROC) plot

Threshold = 0.37

Truth

Indication		p	n
	p	90	35
	n	10	65

Threshold = 2.37

Truth

Indication		p	n
	p	20	0
	n	80	100

Threshold = 1.16

Truth

| Indication		p	n
	p	60	10
	n	40	90

Application to Forensics

- Glass fragments
- Statistical methods of evaluating evidence

# BACKGROUND

History and Uses of ROC Curves

- 1940s: Radar hits and misses
- 1950s and 1960s: Signal detection theory (Green and Swets, 1966)
- 1980s: Diagnostic systems (Swets and Pickett, 1982)
- Today: Medicine, machine learning, astronomy and more

Overview of ROC Curves
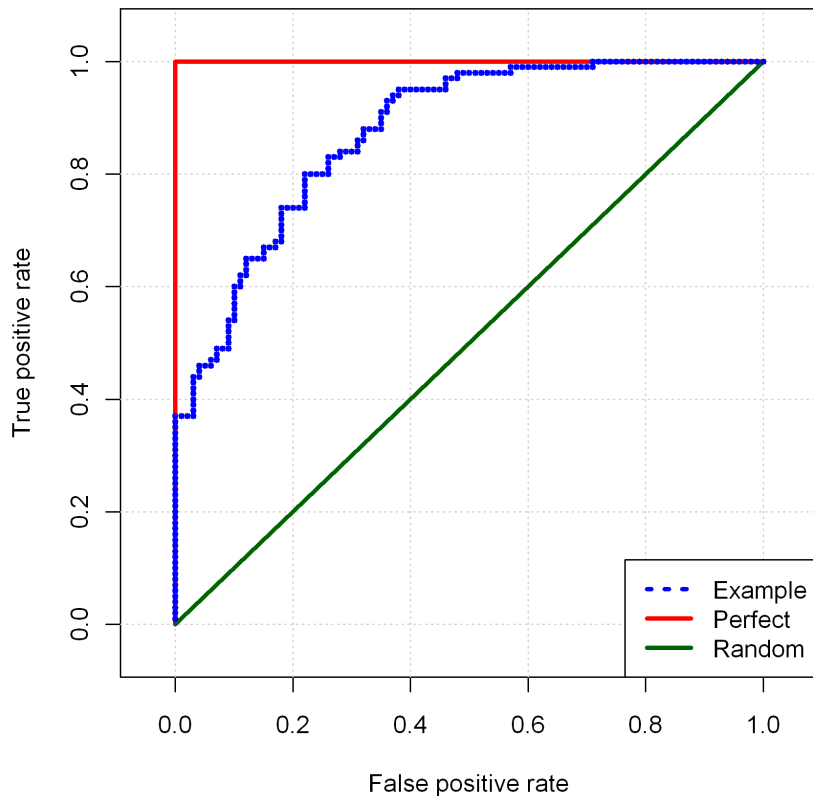
- Axes
  - True positive rate vs false positive rate
  - Range from 0 to 1
- Possible ROC curves
  - Perfect from (0,0) to (0,1) to (1,1).
  - Random along 45 degree line
  - In practice, usually somewhere between

Properties of ROC Curves
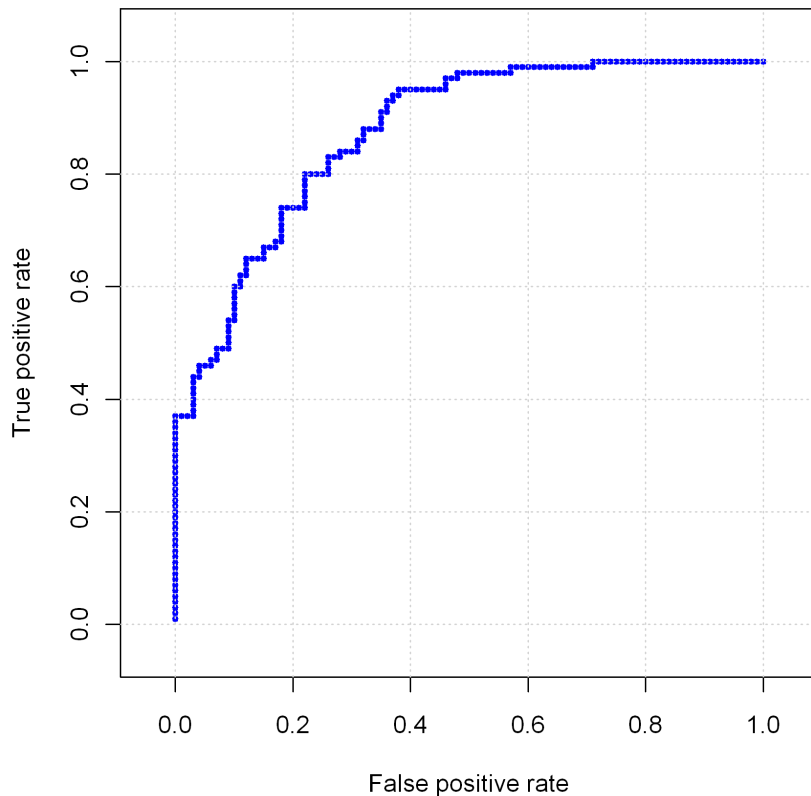
- Complete range of error rates

- Independent of scale of similarity scores
  - Order of similarity scores determines curve
  - Invariant under non-decreasing monotone transformation

ROC Performance Measures
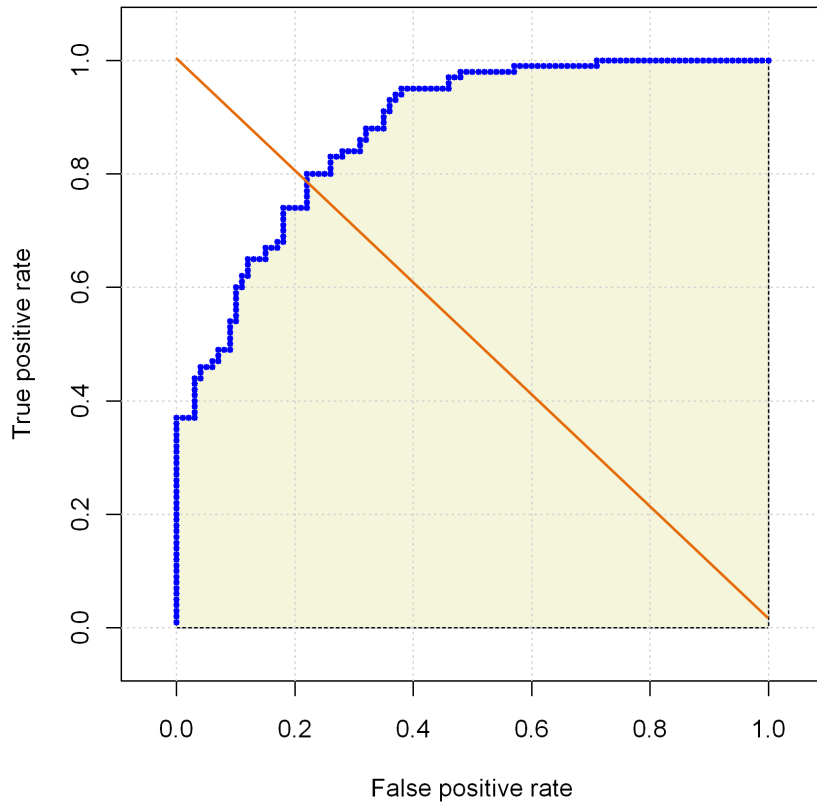
- Error rates (upper left region)

- Equal error rate

- Equal likelihood

- Area under the curve (AUC)

$$= \Pr(\text{score}_{same} > \text{score}_{diff})$$

ANALYSIS

Glass data

- 62 windows
  - Three types
- Five fragments from each window
- Measurements of Si, K, Ca, and Fe
- Variables: log(Ca/K), log(Ca/Si), log(Ca/Fe)

Aitken, C. G. G., and Lucy, D. (January 2004). Evaluation of trace evidence in the form of multivariate data. *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 53 (1), 109-122.

Data publicly available at
http://www.blackwellpublishing.com/rss/

Statistical Methods of Evaluating Evidence

- Methods (Aitken and Lucy, 2004):
  - Multiple t-statistics
  - Hotelling's T^2-statistic
  - Normal-based likelihood ratio
  - Density-based likelihood ratio
- Similarity scores:

  We can treat all methods as mappings from two samples to a similarity score.
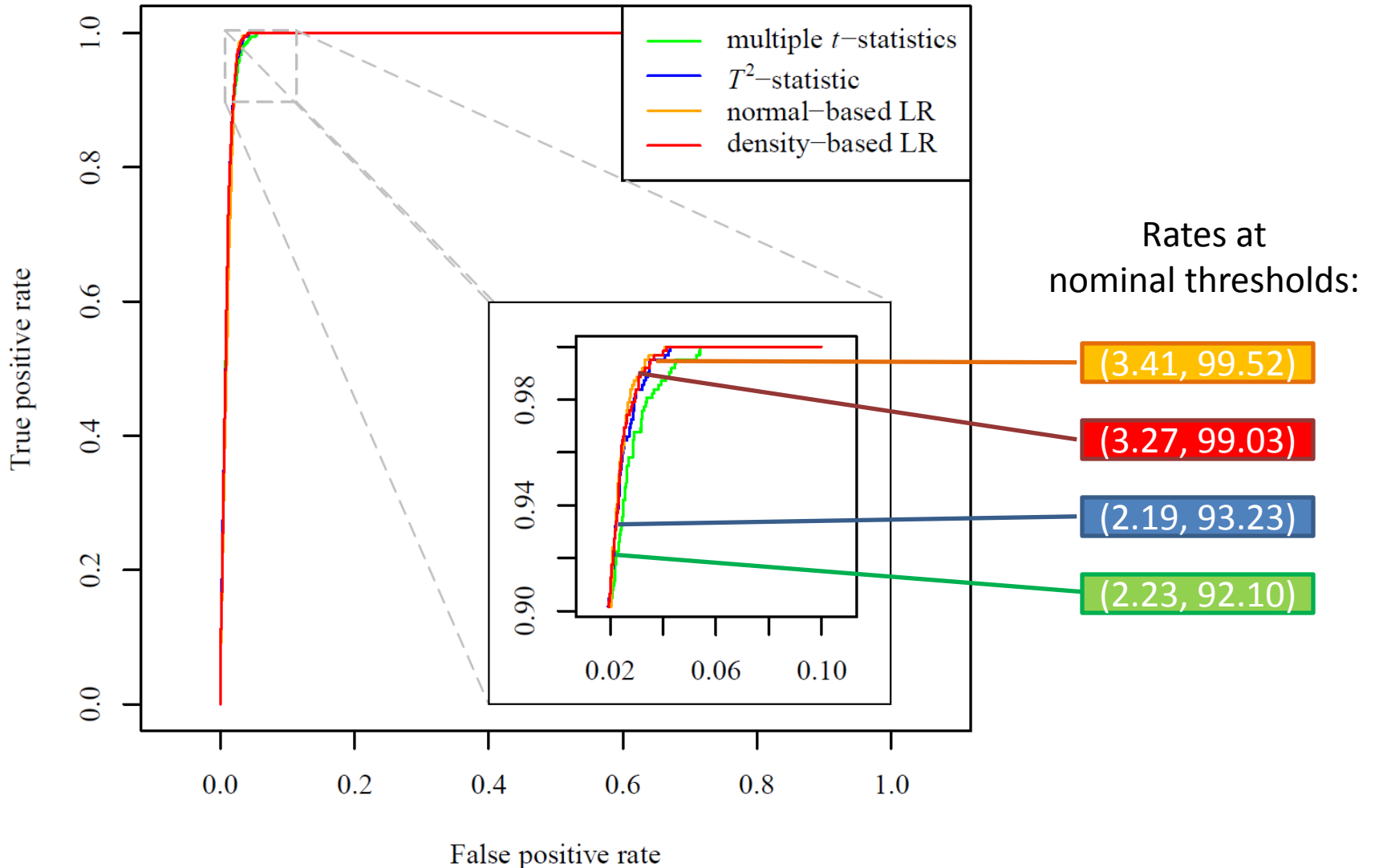
RESULTS

Nominal Error Rates

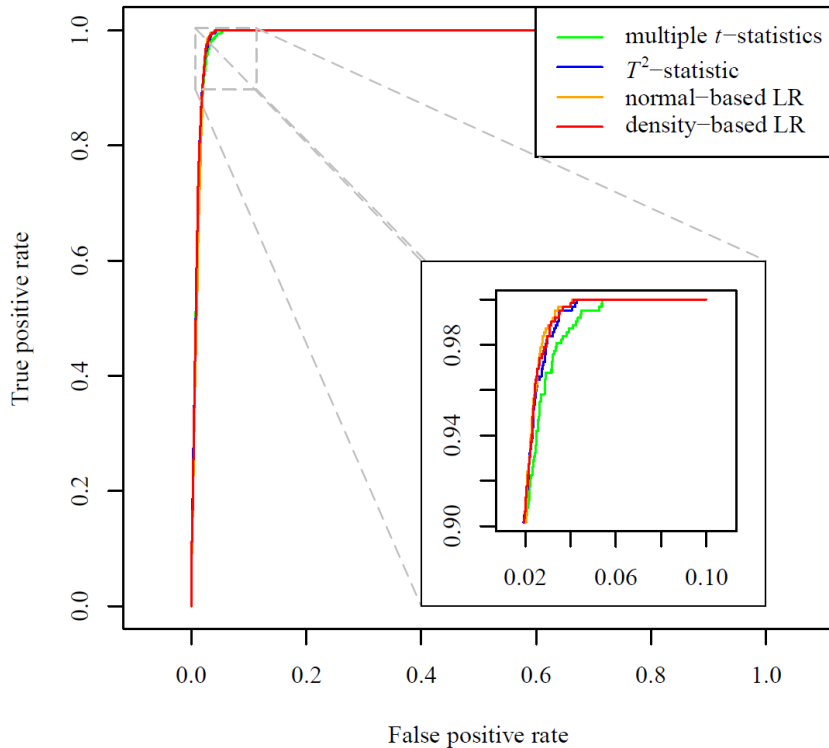Error rates at nominal thresholds for two versus three fragments:

Error type	Multiple t-statistics	T^2-statistic	Normal-based LR	Density-based LR
false negative	7.90	6.77	0.48	0.97
false positive	2.23	2.19	3.41	3.27

ROC Plots

ROC Plot for Two Versus Three Fragments

Legend:
- multiple t-statistics (green)
- T^2-statistic (blue)
- normal-based LR (orange)
- density-based LR (red)

Rates at nominal thresholds:
- (3.41, 99.52)
- (3.27, 99.03)
- (2.19, 93.23)
- (2.23, 92.10)

AUC Values

AUC values

Method	AUC
Multiple t-statistics	99.03
T^2-statistic	99.08
Normal-based LR	98.98
Density-based LR	99.09

EER Values
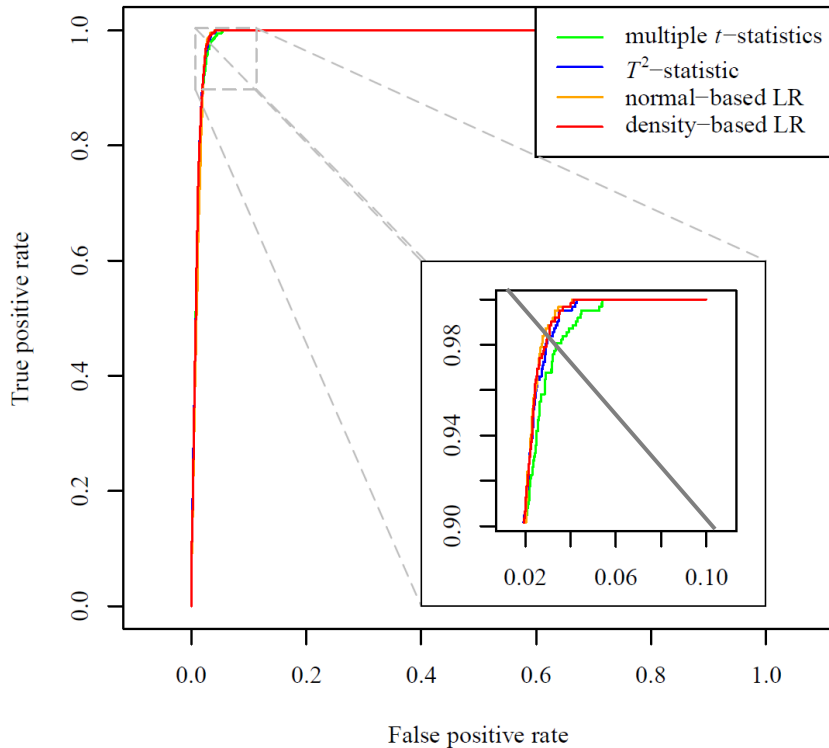
EER values

Method	EER	Threshold
Multiple t-statistics	3.23	3.29
T^2-statistic	2.81	12.44
Normal-based LR	2.61	38.32
Density-based LR	2.62	17.98

5% False Negative Rate
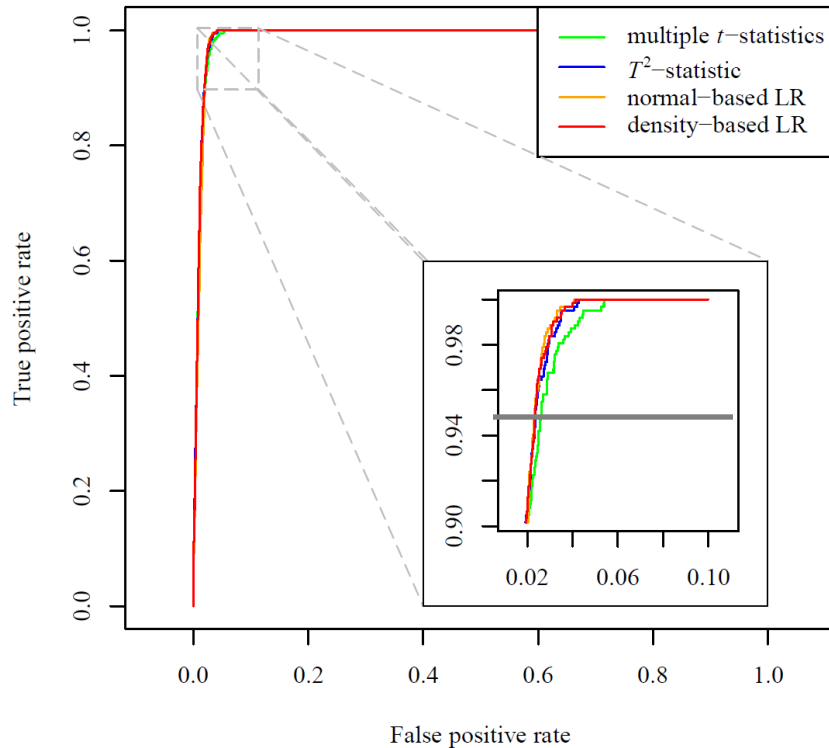
False Positive Rates

Method	False Positive Rate	Threshold
Multiple t-statistics	2.61	2.72
T^2-statistic	2.36	8.96
Normal-based LR	2.30	117.33
Density-based LR	2.35	44.71

CONCLUSION

Conclusions

- ROC curves from similarity scores are
  - Comprehensive
    - Full range of error rates
  - Comparable
    - Independent of scale of scores
    - Objective performance measures

- Application to trace evidence and statistical methods (glass data) showed high performance from all methods.

References

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- Zou, K. H., A. J. O'Malley, and L. Mauri (2007). Receiver-operating characteristic analysis for evaluating diagnostic tests and predictive models. *Circulation* 115 (5), 654–657.